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Second Semester B.E. Degree Examination, Jan./Feb. 2021 Engineering Mathematics – II

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Solve $\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} - 6y = e^{3x}$ (06 Marks)

b. Solve $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 3\sin x$ (07 Marks)

c. Solve by the method of undetermined coefficients

$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 4y = 2x^2 + 3e^{-x}$ (07 Marks)

OR

2 a. Solve $(D^4 + 4D^3 - 5D^2 - 36D - 36)y = 0$ (06 Marks)

b. Solve $\frac{d^2y}{dx^2} + \frac{dy}{dx} = x^2 + 2x + 4$ (07 Marks)

c. Solve by variation of parameters method $\frac{d^2y}{dx^2} + a^2y = \tan ax$ (07 Marks)

Module-2

3 a. Solve $x^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + y = \log x$ (06 Marks)

b. Solve $xy \left(\frac{dy}{dx}\right)^2 - (x^2 + y^2) \frac{dy}{dx} + xy = 0$ (07 Marks)

c. Find the general and singular solution of $y = px - \sin^{-1} p$. (07 Marks)

OR

4 a. Solve $(1+x)^2 \frac{d^2y}{dx^2} + (1+x) \frac{dy}{dx} + y = \sin(2 \log(1+x))$ (06 Marks)

b. Solve $p^2 + 2py \cot x = y^2$, where $p = \frac{dy}{dx}$ (07 Marks)

c. Solve $(px - y)(py + x) = a^2 p$ by taking $x^2 = X$ and $y^2 = Y$. (07 Marks)

Module-3

5 a. Form the partial differential equation from $xyz = \phi(x + y + z)$ (06 Marks)

b. Solve $\frac{\partial^3 z}{\partial x^2 \partial y} + 18xy^2 + \sin(2x - y) = 0$ by direct integration. (07 Marks)

c. Find all possible solutions of the one-dimensional heat equation $U_t = c^2 U_{xx}$ by the method of separation of variables. (07 Marks)

OR

- 6 a. Form the partial differential equation from $z = f(x + at) + g(x - at)$, where a is a constant. (06 Marks)
- b. Solve $\frac{\partial^2 z}{\partial x^2} = a^2 z$, given that at $x = 0$, $\frac{\partial z}{\partial x} = a \sin y$ and $z = 0$. (07 Marks)
- c. With suitable assumptions, derive the one dimensional wave equation as $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ (07 Marks)

Module-4

- 7 a. Evaluate $\int_0^1 \int_0^{\sqrt{1-y^2}} x^3 y dx dy$ by changing the order of integration. (06 Marks)
- b. Evaluate $\int_{-1}^1 \int_{x-z}^{x+z} \int_0^z (x+y+z) dx dy dz$ (07 Marks)
- c. Show that $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ (07 Marks)

OR

- 8 a. Evaluate $\int_0^a \int_0^a \frac{x}{x^2 + y^2} dx dy$ by changing to polar coordinates. (06 Marks)
- b. Using double integration find the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$. (07 Marks)
- c. Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$ (07 Marks)

Module-5

- 9 a. Find Laplace transform of $t(\sin at + \cos at)$ (06 Marks)
- b. Find the Laplace transform of the periodic function of period $2a$ given by

$$f(t) = \begin{cases} t, & 0 < t < a \\ 2a - t, & a < t < 2a \end{cases}$$
 (07 Marks)
- c. Using convolution theorem find $L^{-1}\left[\frac{s}{(s^2 + a^2)^2}\right]$ (07 Marks)

OR

- 10 a. Express $f(t) = \begin{cases} \cos t, & 0 < t < \pi \\ \cos 2t, & \pi < t < 2\pi \\ \cos 3t, & t > 2\pi \end{cases}$ in terms of unit-step function and hence find $L(f(t))$. (06 Marks)
- b. Find the inverse Laplace transform of
 i) $\frac{s^2 - 3s + 4}{s^3}$ and ii) $\frac{s + 2}{s^2 - 4s + 13}$ (07 Marks)
- c. Solve by Laplace transform method $\frac{d^2 x}{dt^2} - 2 \frac{dx}{dt} + x = e^t$ with $x = 2$, $\frac{dx}{dt} = -1$ at $t = 0$. (07 Marks)
